

# Comment on “On the Electric Charge Quantization from the Aharonov-Bohm Potential”

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## Abstract

In the paper [1], Barone and Halayel-Neto (BH) claim that charge quantization in quantum mechanics can be proven without the need for the existence of magnetic monopoles. In this paper it is argued that their claim is untrue.

In the paper [1], Barone and Halayel-Neto (BH) claim that charge quantization in quantum mechanics can be proven without the need for the existence of magnetic monopoles. The argument relies on a re-analysis of the Aharonov-Bohm (AB) effect, as follows. The magnetic field of an infinitely long solenoid of radius  $R$  lying along the  $z$  axis is

$$\mathbf{B}_{\text{sol}}(\mathbf{r}) = \hat{z}B\Theta(R - \rho), \quad (1)$$

where  $\Theta(x)$  is the Heaviside step function, equal to 0 or 1 for  $x$  negative or positive, respectively. This is described in [1] by the vector potential

$$\mathbf{A}(\mathbf{r}) = \hat{\phi} \left\{ \frac{B\rho}{2}\Theta(R - \rho) + \frac{\gamma}{2\rho}\Theta(\rho - R) \right\}, \quad (2)$$

where  $\gamma$  is, a priori, a free constant.

Were  $\gamma = BR^2/2$  (the standard choice in the literature, written  $\gamma_{\text{AB}}$  in [1]), this vector potential indeed describes  $\mathbf{B}_{\text{sol}}$ ; with any other value of  $\gamma$  there is (as noted in [1]) in addition a magnetic field localized at  $\rho = R$  whose integrated flux is  $\Phi' = 2\pi\gamma - \Phi_{\text{sol}}$ , where  $\Phi_{\text{sol}} = B\pi R^2$  is the flux of the solenoid. BH describe the cylinder  $\rho = R$  as “non-physical,” possibly because it is a “singularity region for the field strength, since there is a surface charge density,” which apparently justifies their use of a vector potential describing a magnetic flux (as measured by  $\int \mathbf{A} \cdot d\mathbf{l}$  integrated around a curve encircling the solenoid) unequal to  $\Phi_{\text{sol}}$ . BH parameterize their choice of vector potential by  $\kappa \equiv \gamma - \gamma_{\text{AB}}$ , so that  $\Phi' = 2\pi\kappa$ .

It is then argued that the solenoid will exhibit the usual AB effect (with the correct magnetic flux,  $\Phi_{\text{sol}}$ ) for particles of charge  $q_i$  only if  $q_i\kappa \in \mathbf{Z}$ , which is to be viewed as a condition on  $\kappa$ . In order for this to occur for a particle of charge  $q_1$ , we must therefore have  $\kappa = n_1/q_1$ , where  $n_1$  is some integer. A second charged particle, of charge  $q_2$ , must then obey  $q_2\kappa = n_2$ ,  $n_2 \in \mathbf{Z}$ , so that

$$q_2 = \frac{n_2}{n_1}q_1. \quad (3)$$

This is the charge quantization condition as derived by BH.

Our objections can be put into two categories. Firstly, and most bluntly, the vector potential (2) is simply wrong. Rather than declaring the position of the solenoid to be “unphysical”, since indeed  $B_{\text{sol}}$  is discontinuous there, and so ignoring the flux in that region, one should recognize that (1) is the field of an ideal solenoid made of infinitely thin wires infinitely close to one another. A better treatment of this “singularity region” would be to smooth out the discontinuity in the magnetic field (equivalent to

considering finite-thickness wires) so that it is equal to  $B$  inside a certain radius and zero outside a second slightly larger radius, with a smooth interpolation between these two values. Such a magnetic field can easily be described unambiguously (up to gauge transformation) by a smooth vector potential without invoking fictitious magnetic fluxes which must subsequently be rendered unobservable by insisting on a charge quantization condition.

But let us put aside this argument and examine the reasoning used in [1], and its consequences. Starting with the consequences, their main result (3) is an exceedingly weak charge quantization condition, indeed. It merely states that all charges must be related to one another by rational factors. To illustrate the weakness of (3), note that while it must be admitted that charges  $e$  and  $\sqrt{2}e$  are not compatible,  $e$  and  $1.4142136e$  are. (The reader may replace  $\sqrt{2}$  by her/his favourite irrational number, and  $1.4142136$  by an arbitrarily accurate rational approximation to it.)

What of the reasoning itself? Essentially, the authors let the value of one charge determine the possible values of  $\kappa$ , which then determines the possible values of all other charges in the problem, by insisting that the flux  $\Phi'$  causes no AB effect for any particle.

(While somewhat peripheral to the discussion at hand, note that this seems to select one charge [the one which determines the allowed values of  $\kappa$  – that is, the allowed strengths of the fictitious delta-function magnetic field which can be added] as having a special role. Indeed, the allowed strengths of the fictitious magnetic field would be different if this initial charge were  $e$  or  $1.4142136e$ . One might argue that this is not a serious problem, since, after all, it is arranged that the fictitious magnetic field is unobservable. The more important point, perhaps, is that if the initial charge were  $e$ , then  $1.4142136e$  is allowed, and vice versa. Essentially, the choice of with which charge one begins is immaterial in so far as charge quantization is concerned, because (3) is symmetric.)

However, why not turn the argument around and argue that the additional delta-function field must be constrained by insisting that, whatever charges exist, this additional field must yield no AB effect? In other words, rather than having one charge constrain  $\kappa$  and then having the allowed values of  $\kappa$  constrain subsequent charges, why not let the pre-existing charges (which obey no a priori quantization condition) constrain  $\kappa$ ? For instance, if  $e$  and  $1.4142136e$  existed, certain magnetic fields could exist without having any observable effects. (To be specific, writing  $1.4142136 = m/n$  with  $m, n$  relatively prime integers, one finds that the allowed values of  $\kappa$  are integer multiples of  $n/e$ .) However, let us apply this same reasoning to a situation where charges  $e$  and  $\sqrt{2}e$  exist. In order for neither charge to experience an AB effect, it is necessary that  $\kappa$  is an integer multiple of *both*  $1/e$  and  $1/(\sqrt{2}e)$ . This has only one possible solution:  $\kappa = 0$ . Thus, the nonexistence of an AB effect arising from the delta-function magnetic field considered in [1] might be used to eliminate charge non-quantization, but it is equally true that with only a minor change of logic the existence of charge non-quantization provides a second reason for eliminating the fictitious delta-function magnetic field. Since neither reasoning seems advantageous over the other, we prefer to invoke the physical reasoning outlined in our first objection above, wherein the added delta-function magnetic field is eliminated by making the standard choice of gauge potential.

In summary, unless there is an *a priori* reason *independent of whatever charges are present* for restricting the allowed values of the parameter  $\kappa$ , the derivation of a charge quantization condition by insisting that  $\kappa$  be unobservable is firstly without teeth (since the charge quantization condition so derived is so weak), and secondly, not even necessary. Furthermore, a much stronger objection can be raised, in that the addition of an ad hoc delta-function magnetic field, which is at the heart of the charge quantization condition derived in [1], is completely without physical motivation: if the discontinuous magnetic field  $B_{\text{sol}}$  is to be avoided, one need merely smooth it out, as would be the case in any case with a real solenoid. No fictitious magnetic field arises,  $\kappa = 0$ , and the quantization condition – weak as it is – never sees the light of day.

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## References

- [1] F.A. Barone and J.A. Helayel-Neto, “On the Electric Charge Quantization from the Aharonov-Bohm Potential,” quant-ph/0503212. See also F.A. Barone and J.A. Helayel-Neto, “A Remark on the Aharonov-Bohm Potential and a Discussion on the Electric Charge Quantization,” quant-ph/0409074.